Section 6: Percentages

6.1 Percentages

When comparing fractions, it is often convenient to express them with a denominator of 100. Fractions with a denominator of 100 are called **percentages** and the numerator of the fraction is called the **rate per cent**.

Thus $\frac{2}{5} = \frac{40}{100} = 40$ per cent

The symbol % is usually used to denote per cent.

To convert a fraction into a percentage we need to multiply the fraction by 100 per cent.

Thus in the above example, $\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$.

Example:

(i) On a certain day, 4 students from a class of 32 students are absent. What percentage of students are present on that day?

(ii) 8% of the apples in a box containing 75 apples were rotten. How many apples were rotten?

(iii) 6% of the children in a school caught an infectious disease. If the number of children who caught the disease is 48, how many students are there in the school?

Solution:

(i) Let x be the percentage of students that are present. The number of students present is 28. Therefore, $x = \frac{28}{32} \times 100\% = 87.5\%$. Thus 87.5% of the students are present.

(ii) Let x be the number of rotten apples in the box. Then $\frac{x}{75} = \frac{8}{100}$. Thus $x = \frac{75 \times 8}{100} = \frac{3 \times 8}{4} = 6$; i.e., 6 apples were rotten.

(iii) Let x be the number of students in the school. Then $\frac{6}{100} = \frac{48}{x}$. Thus $x = \frac{48 \times 100}{6} = 800$; i.e., the number of students in the school is 800.

6.1.1 Profit and loss

When goods are bought for one sum of money and sold for another, there is a profit or loss depending on whether the selling price is greater or less than the cost price. The **cost price** (C.P) is the sum of money paid to buy the goods. The price at which the goods are sold is the **selling price** (S.P). If the selling price is more than the cost price, then the seller makes a **profit** which is equal to the difference between the selling price and the cost price.

Profit = Selling Price – Cost Price (if the selling price is greater than the cost price)

However if the selling price is less than the cost price, then the seller suffers a **loss** which is equal to the difference between the cost price and the selling price.

Loss = Cost Price - Selling Price (if the cost price is greater than the selling price)

6.1.2 Percentage Profit and Discount

Example:

Consider the following cost prices and respective selling prices of two items.

(i) Cost Price - Rs. 25; Selling Price – Rs. 30
(ii) Cost Price – Rs. 300; Selling Price - Rs. 305

In each of the above cases, the profit is Rs. 5. But, in the first case the profit is $\frac{5}{25} = \frac{1}{5}$ of the cost price while in the second case the profit is $\frac{5}{300} = \frac{1}{60}$ of the cost price.

Expressing these as percentages we obtain,

the profit as a percentage in the first case = $\frac{1}{5} \times 100\% = 20\%$

the profit as a percentage in the second case = $\frac{1}{60} \times 100\% = 1.67\%$

Thus despite the profit being the same in both cases, the rate of profit is different, and the better deal for the seller is the first case.

We see that the calculation of the actual gains or losses does not provide us with any useful basis for comparison, but the profit per cent and the loss per cent do.

We note that the profit per cent has been calculated with reference to the cost price, and unless otherwise specified, profit per cent and loss per cent are always to be understood in this sense.

Thus

Profit per cent = $\dfrac{\textbf{profit}}{\textbf{cost}} \times 100\%$

Loss per cent = $\dfrac{\textbf{loss}}{\textbf{cost}} \times 100\%$

Example:

(i) A salesman buys a wristwatch for Rs. 1100, spends Rs. 250 to repair it and sells it for Rs. 1500. What is his profit per cent?

(ii) Renuka sells a pair of shoes for Rs. 736 by making a profit of 15%. How much did she spend to buy the pair of shoes?

(iii) Ruvan sells an old TV for Rs. 5000 by incurring a loss of 20%. What was the price that he initially bought it for? To have gained a profit of 5%, how much should he have sold it for?

Solution:

(i) Cost price = Rs. 1100 + Rs. 250 = Rs. 1350
Profit = Rs. 1500 – Rs. 1350 = Rs. 150
Thus profit per cent = $\dfrac{150}{1350} \times 100\% = 11\dfrac{1}{9}\%$

(ii) If the cost of the pair of shoes was Rs. x, then $x + \dfrac{15}{100}x = 736$.
Therefore $x(1+\dfrac{3}{20}) = 736$; i.e., $\dfrac{23}{20}x = 736$.
Therefore $x = \dfrac{736 \times 20}{23} = 32 \times 20 = 640$.
i.e., Renuka bought the pair of shoes for Rs. 640.

(iii) If the price that Ruvan bought the TV for is Rs. x, then since
"Selling Price = Cost Price – Loss, we obtain $5000 = x - \dfrac{20}{100} \times x$.
i.e., $5000 = \dfrac{80}{100} \times x$. Therefore $x = 5000 \times \dfrac{100}{80} = 6250$. i.e., the price he initially bought it for is Rs. 6250.

Since "Selling Price = Cost Price + Profit", to have gained a profit of 5%, he should have sold it for

Rs. 6250 + 6250 $\times \frac{5}{100}$ = Rs. 6250 $\times \frac{105}{100}$ = Rs. 6562.50.

6.1.3 Discounts

Shop keepers at times have sales to get rid of old stocks. At such times goods are sold at prices that are lower than the price on the 'tag' which is the **marked price**. The shop keeper usually still makes a profit, though less than what he would have made if he sold the item for the marked price.

Example (Challenging Problems):

(i) A shop keeper gives a discount of 10% on the marked price of a shirt and still makes a profit of 25%. If the marked price is Rs. 500, what was the actual cost of the shirt?

(ii) Kamal buys a pair of shoes on sale at a discount of 20%. He sells it for Rs. 880, making a profit of 10%. What are the marked price and the discount price?

Solution:

(i) The marked price of the shirt is Rs. 500.
Since "Selling Price = Marked Price – Discount", the selling price of the shirt with the discount is Rs. (500 – 500 $\times \frac{10}{100}$) = Rs. 500 $\times \frac{90}{100}$ = Rs. 450.
If the actual cost of the shirt is Rs. x, then since "Selling Price = Cost Price + Profit", 450 = $x + x \times \frac{25}{100} = x \times \frac{125}{100}$. Thus x = 450 $\times \frac{100}{125}$ = 360; i.e., the actual cost of the shirt was Rs. 360.

(ii) Suppose Kamal buys the pair of shoes for Rs. x.
Selling Price = Cost Price + Profit. Since Kamal makes a profit of 10% by selling the pair of shoes for Rs. 880 we obtain, 880 = $x + x \times \frac{10}{100} = x \times \frac{110}{100}$.
Thus x = 880 $\times \frac{100}{110}$ = 800; i.e., the discount price is Rs. 800.
Suppose the marked price is y.
Then since, "Selling Price = Marked Price – Discount",
800 = $y - \frac{20}{100} \times y = \frac{80}{100} \times y$.
Thus the marked price y = Rs. $\frac{800 \times 100}{80}$ = Rs. 1000.

6.2 Simple interest and compound interest

People borrow money from money lenders, banks and other financial institutions for various requirements. The borrowed money is usually paid back after a certain period, with an additional amount which is the charge for the money lent. Such a charge is called the **interest**, and the sum lent is called the **principal**. Money is also invested in banks and other financial institutions for interest.

Interest is usually given as a percentage on the principal for each year until the loan (or deposit) is repaid. The sum paid on each Rs. 100 of the loan (or deposit) for each year is called the **rate per cent per annum**.

When the interest is paid on the original principal only, throughout the whole term of the loan (or deposit), it is called **simple interest**. When interest, as it becomes due is used to increase the principal, the interest is said to be **compound**.

The principal together with its interest for the stated time is called the **amount** for that time.

6.2.1 Simple Interest

Simple interest depends on

(i) the amount borrowed, lent or deposited
(ii) the rate per cent
(iii) the period of the loan or deposit

When the principal P, rate per cent per annum r, and the period of the loan or deposit T (in years) are given, the simple interest can be calculated by the formula

$$I = \frac{PrT}{100}$$

Example:

(i) What is the simple interest on a loan of Rs. 5000, invested for 3 years at a rate of 9% per annum? What is the amount payable after 3 years?

(ii) If the simple interest on a loan after 2 years is Rs. 2400 calculated at a rate of 12 per cent per annum, what is the principal amount?

(iii) How many years would it take a deposit of Rs. 1000 to accumulate an interest of Rs. 300, if simple interest is paid at a rate of 12% per annum?

Solution:

(i) $I = \frac{5000 \times 9 \times 3}{100} = 1350$. Thus the simple interest is Rs. 1350 and the amount payable after 3 years is Rs. 5 000 + Rs. 1 350 = Rs. 6 350.

(ii) $2400 = \frac{P \times 12 \times 2}{100}$. Thus P = Rs. $\frac{2400 \times 100}{24}$ = Rs.10 000.

(iii) $300 = \frac{1000 \times 12 \times T}{100}$. Thus $T = \frac{300 \times 100}{1000 \times 12} = 2.5$ years.

6.2.2 Compound Interest

Money is said to be invested (or loaned out) at **compound interest** when each instalment of interest as it becomes due is added to the principal. In this case the principal is continually being increased and the interest for each period is the interest on the amount at the end of the preceding period.

Example:

Suppose Rs. 1000 is invested at an annual compound interest rate of 10% for 3 years. Then, at the end of the first year, the interest is Rs. 100. Thus the principal for the second year is Rs. 1100. Therefore the interest at the end of the second year is Rs. 110 and the principal amount for the third year is Rs. 1100 + Rs. 110 = Rs. 1210. Thus the interest for the third year is Rs. 121. Therefore the amount at the end of three years is Rs. 1 331 and the total compound interest paid is Rs. 1331 – Rs. 1000 = Rs. 331.

Note: If Rs. 1000 was invested for 3 years at a simple interest rate of 10%, then the interest accrued after 3 years would be Rs. 300. Thus compound interest is beneficial for investors, but not for borrowers.

When the principal P, rate per cent per annum r, and the period of the loan or deposit t (in years) are given, the amount accrued after t years when interest is compounded can be calculated by the formula

$$A = P\left(1 + \frac{r}{100}\right)^t$$

Sometimes interest is paid half yearly or quarterly. In such cases, each half year or quarter is considered as a separate period at the corresponding interest rate.

Therefore, if the principal P is invested (or borrowed) at a rate per cent per annum r for a period of t years, and interest is compounded n times per year, then the amount accrued after t years can be calculated by the formula

$$A = P\left(1+\frac{r}{100n}\right)^{nt}$$

Example:

(i) If Rs. 2000 is invested in an account paying compound interest at a rate of 12% per annum, what is the value of the investment to the nearest rupee after 3 years?

(ii) If Rs. 2000 is invested for 2 years at a rate of 10% per annum compounded semi-annually, how much interest would have accrued by the end of the two years?

Solution:

(i) $A = 2000(1+0.12)^3 = 2809.856$

Therefore, the value of the investment to the nearest rupee after three years is Rs. 2810.

(ii) We apply the formula $A = P\left(1+\frac{r}{100n}\right)^{nt}$.

P = 2000, r = 10, n = 2 and t = 2. Therefore, $A = 2000(1+0.05)^4 = 2431.0125$.

Thus the amount of the investment after two years is Rs. 2431 to the nearest rupee. Thus the interest accrued is Rs. 431.