Section 4: Fundamentals of Measurements

4.1 Measurements of length, mass, capacity, area and volume

Measurements of length
The **metre** is the international standard unit of length.

Table of Conversion (Metric)

10 millimetres (mm)	= 1 centimetre (cm)
10 centimetres	= 1 decimetre (dm)
10 decimetres	= 1 **metre** (m)
10 metres	= 1 decametre (dam)
10 decametres	= 1 hectometre (hm)
10 hectometres	= 1 kilometre (km)

Note: 100 centimetres = 1 metre,
1000 metres = 1 kilometre

Short lengths are measured in millimetres, centimetres and metres. Long distances are measured in metres and kilometres.

Measurements of mass
The **gram** is the international standard unit of mass.

Table of Conversion (Metric)

10 milligrams (mg)	= 1 centigram (cg)
10 centigrams	= 1 decigram (dg)
10 decigrams	= 1 **gram** (g)
10 grams	= 1 decagram (dag)
10 decagrams	= 1 hectogram (hg)
10 hectograms	= 1 kilogram (kg)

Note: 1000 milligrams = 1 gram
1000 grams = 1 kilogram

Measurements of capacity
The **litre** is the international standard unit of capacity.

Table of Conversion (Metric)

10 millilitres (ml)	= 1 centilitre (cl)
10 centilitres	= 1 decilitre (dl)
10 decilitres	= 1 **litre** (l)
10 litres	= 1 decalitre (dal)
10 decalitres	= 1 hectolitre (hl)
10 hectolitres	= 1 kilolitre (kl)

Note: (i) 1000 millilitres = 1 litre.
(ii) These measurements are used for liquids.

A quantity expressed in a single unit is said to be **simple**. A quantity expressed in two or more units is said to be **compound**.

Example: 7 km is a simple quantity. 200m 20cm is a compound quantity.

Example: Write down in compound quantities

1. 7022 mm
2. 5173 g (in kg and g)

Solution:

1. 7022 mm = 7 m 2 cm 2 mm
2. 5173 g = 5 kg 173 g

Measurements of area

A **square centimetre** ($1cm^2$) is the region enclosed within a square with sides of length 1 cm each. Similarly, a **square metre** ($1\ m^2$) is the region enclosed within a square with sides of length 1 m each.

Table of Conversion (Metric)

100 square millimetres (mm^2)	= 1 square centimetre (cm^2)
100 square centimetres	= 1 square decimetre (dm^2)
100 square decimetres	= 1 **square metre** (m^2)
100 square metres	= 1 square decametre (dam^2)
100 square decametres	= 1 square hectometre (hm^2)
100 square hectometres	= 1 square kilometre (km^2)

Measurements of volume

A **cubic centimetre** is the amount of space enclosed within a cube with sides of length 1 cm each. Similarly, a **cubic metre** is the amount of space enclosed within a cube with sides of length 1 m each.

Table of Conversion (Metric)

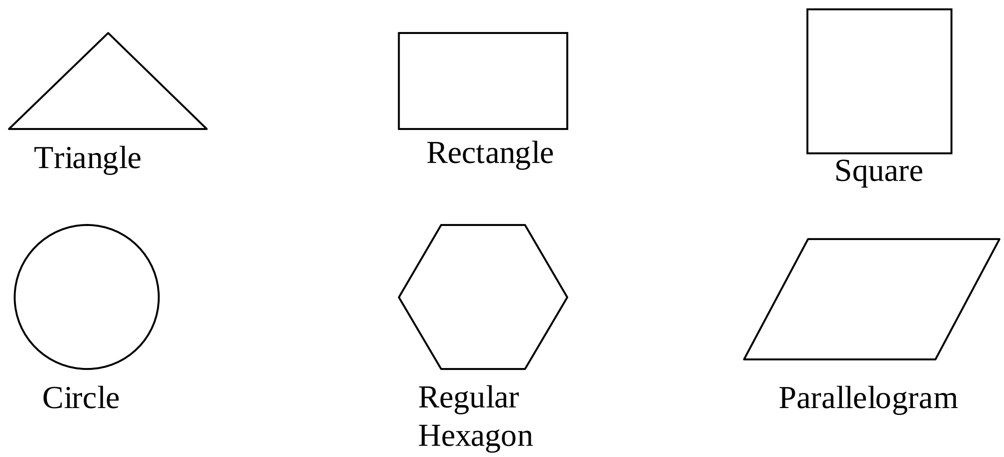
1000 cubic millimetres (mm^3)	= 1 cubic centimetre (cm^3)
1000 cubic centimetres	= 1 cubic decimetre (dm^3)
1000 cubic decimetres	= 1 cubic **metre** (m^3)
1000 cubic metres	= 1 cubic decametre (dam^3)
1000 cubic decametres	= 1 cubic hectometre (hm^3)
1000 cubic hectometres	= 1 cubic kilometre (km^3)

4.2 Perimeter, area and volume of basic geometrical shapes

4.2.1 Plane figures

A closed **plane figure** is a two-dimensional shape which is bounded by lines (straight or curved).

Example:



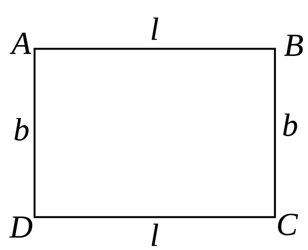
Plane figures which are bounded by straight line segments are called **polygons.** The straight line segments are called the **sides** of the polygon.
A **regular polygon** is one which has all its sides the same length and all its interior angles the same magnitude.
A polygon with three sides is called a **triangle**.
A polygon with four sides is called a **quadrilateral**.

4.2.2 Perimeter and area of plane figures

The **perimeter** of a plane figure is the total length of its boundary.

The **area** is a measure of the surface covered by a given shape; i.e., it is the amount of surface enclosed within its bounding lines.

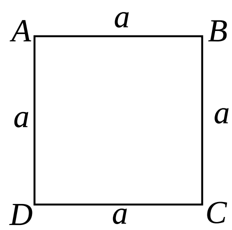
Rectangle



The **perimeter** ***P*** of a **rectangle** of **length** ***l*** and **breadth** ***b*** is given by the formula $\boldsymbol{P = 2(l + b)}$**;** i.e., $\boldsymbol{P = 2l + 2b}$**.**

The **area** ***A*** of a **rectangle** of **length** ***l*** and **breadth** ***b*** is given by the formula $\boldsymbol{A = l \times b}$.

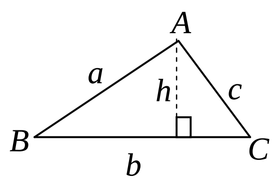
Square



The **perimeter** ***P*** of a **square** with **equal sides of length** ***a*** is given by the formula $\boldsymbol{P = 4a}$.

The **area** ***A*** of a **square** with **equal sides of length** ***a*** is given by the formula $\boldsymbol{A = a^2}$.
A square with a = 1 cm, has area equal to 1 square centimetre (1 cm^2)
A square with a = 1 m has area equal to 1 square metre (1 m^2)

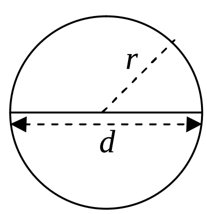
Triangle



The **perimeter** ***P*** of a **triangle** with **sides of length** ***a*, *b*** and ***c*** respectively is given by the formula $\boldsymbol{P = a + b + c}$**.**

The **area** ***A*** **of a triangle** with **base length** ***b*** and **altitude** ***h*** (perpendicular height) is given by the formula $\boldsymbol{A} = \frac{1}{2} \times \boldsymbol{b} \times \boldsymbol{h}$.

Circle



The **perimeter** of a **circle** is called the **circumference** of the circle. The boundary of a circle is also called its circumference.

A section of the circumference is called an **arc**.
A **chord** is a line joining two points on the circumference.
A **diameter** is a chord which passes through the centre of the circle.
A **segment** is a part of a circle cut off by a chord.
A **sector** is a part of a circle cut off by two radii.

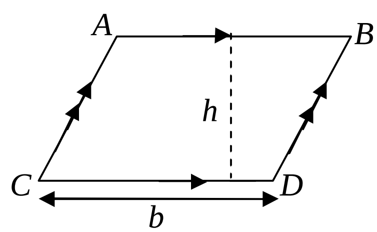
The **circumference** $\boldsymbol{C}$ of a **circle** of **radius** $\boldsymbol{r}$ is given by the formula $\boldsymbol{C = 2\pi r}$.
Here π is an irrational number which is approximately equal to 3.14 or $\frac{22}{7}$.

The **diameter** $\boldsymbol{d}$ of a circle equals **2 × radius**. Therefore the circumference is also given by the formula $\boldsymbol{C = \pi d}$.
The **area** $\boldsymbol{A}$ of a **circle** of **radius** $\boldsymbol{r}$ is given by the formula $\boldsymbol{A = \pi r^2}$

Thus the area A of a semi-circle of radius r is given by the formula $A = \dfrac{\pi r^2}{2}$.

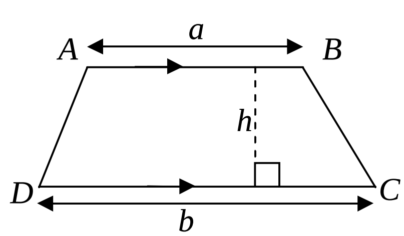
Parallelogram

A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.



The **area** $\boldsymbol{A}$ of a **parallelogram** of **base length** $\boldsymbol{b}$ and **altitude** $\boldsymbol{h}$ is given by
$\boldsymbol{A = b \times h}$

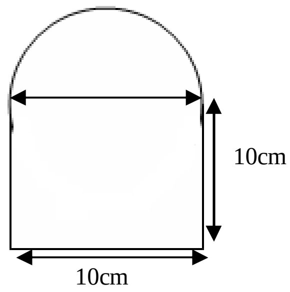
Trapezium



The **area** $\boldsymbol{A}$ of a **trapezium** with **parallel sides of lengths** $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively and **altitude** $\boldsymbol{h}$ is given by $\boldsymbol{A} = \frac{1}{2}(a+b)h$.

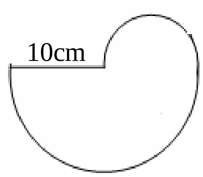
Example:
Find the perimeter and area of the following figures.

(a) Compound plane figure of a square and a semi-circle



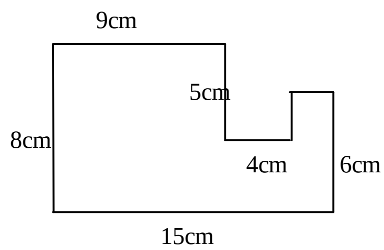
Solution:
Perimeter: [10 + 10 + 10 + 5π] cm = (30 + 5π) cm
Area: $[10 \times 10 + \frac{1}{2}\pi (5^2)]\ \text{cm}^2 = (100 + 12.5\pi)\ \text{cm}^2$

(b) Compound plane figure of two semi-circles as in the figure below, where the radius of the larger circle is 10cm



Solution:
Perimeter: [10 + 10π + 5π] cm = (10 + 15π) cm
Area: $[\frac{1}{2}\pi(10^2) + \frac{1}{2}\pi(5^2)]\ \text{cm}^2 = \frac{125}{2}\pi\ \text{cm}^2$

(c)



Solution:
Perimeter: [15 + 8 + 9 + 5 + 4 + 3 + 2 + 6] cm = 52cm
Area: $[(8\times9) + (4\times3) + (6\times2)]\ \text{cm}^2 = 96\ \text{cm}^2$

4.2.3 Solids

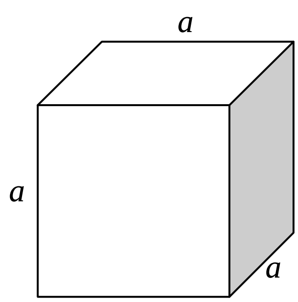
A **solid** is a three-dimensional shape.
A **polyhedron** is a solid bounded by planes.
A **polyhedral angle** is formed when three or more planes meet at a common point called a **vertex**. The intersections of the planes are called **edges**. The sections of the planes between the edges are called **faces**.
A **regular polyhedron** has faces which are congruent (identical) regular polygons and equal polyhedral angles.

4.2.4 Surface area and volume of solids

The **volume** of a solid is the measure of the amount of space which is taken up by the solid shape.

Cube

A **cube** is a solid with six square faces (or sides). Hence the length, breadth and height of a cube are all equal.



The **volume** of a **cube** = **length × breadth × height**
The **surface area** of a **cube** = **6 × area of a face**

The **surface area of a cube** with **length, breadth and height** equal to ***a* cm** is equal to $\mathbf{6a^2}$ $\mathbf{cm^2}$ (since the area of each face is a^2 cm^2 and there are 6 such faces).

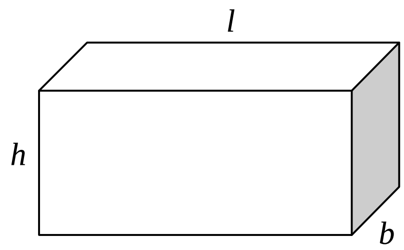
The **volume of a cube** with **length, breadth and height** equal to ***a* cm** is equal to $\mathbf{a^3}$ $\mathbf{cm^3}$.

Relationship between volume and capacity

1000 cubic centimetres is equivalent to 1 litre

Cuboid

A **cuboid** is a solid with six rectangular faces.



The **volume of a cuboid** = **length** × **breadth** × **height**

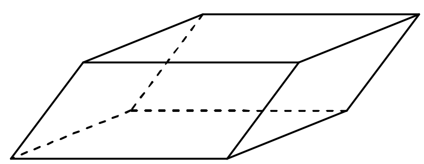
For a cuboid of length l, breadth b and height h,
Volume = $\boldsymbol{l \times b \times h}$
Surface Area = $\boldsymbol{2(l \times b) + 2(b \times h) + 2(l \times h)}$

Prisms

A **prism** is a polyhedron, two faces of which are congruent polygons in parallel planes called the **bases** of the prism. The other faces are called the **lateral faces** of the prism. If the lateral faces are perpendicular to the bases, the prism is called a **right prism**. A **regular prism** is a prism which has regular polygons as bases. The **altitude** of a prism is the perpendicular distance between its bases.

Parallelepiped

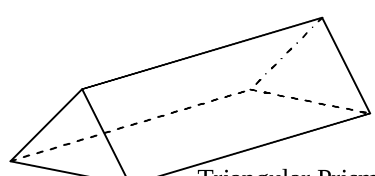
A **parallelepiped** is a prism with parallelograms as bases.



A rectangular prism or right parallelepiped is a cuboid. A cube is a right parallelepiped in which all the six faces are congruent squares.

Triangular Prism

A **triangular prism** is a prism with triangles as bases.



Triangular Prism

Surface area of a prism

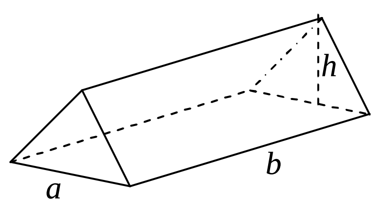
The **lateral area** of a prism is the sum of the areas of the lateral faces. The **lateral area** $\boldsymbol{S}$ of a **right prism** equals the **product** of the **base perimeter** $\boldsymbol{p}$ and the **altitude** $\boldsymbol{h}$; $\boldsymbol{S = p \times h}$**.**

The **surface area** $\boldsymbol{A}$ of a prism is the **sum** of the **lateral area** $\boldsymbol{S}$ and the **areas** $\boldsymbol{B_1}$ **and** $\boldsymbol{B_2}$ **of the two bases**; $\boldsymbol{A = S + B_1 + B_2}$**.**

Volume of a prism

The **volume** $\boldsymbol{V}$ of a prism equals the **product** of its **base area** $\boldsymbol{B}$ and its **altitude** $\boldsymbol{h}$; $\boldsymbol{V = B \times h}$**.**

Example:

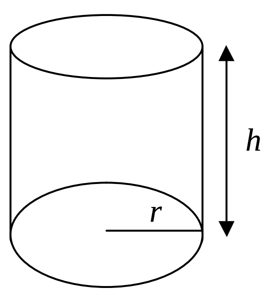


Right Triangular Prism

Volume of the Prism**:** $(\frac{1}{2}\times a\times h)\times b$. Here the altitude is b and the base area is $\frac{1}{2}\times a\times h$.

Cylinder

A **right circular cylinder** is bounded by two parallel planes (bases) and by a surface generated by revolving a rectangle about one of its sides. The **bases** of a right circular cylinder are **circles**. The **axis** of a right circular cylinder is the line joining the centres of the circular bases. The **altitude** $\boldsymbol{h}$ of a cylinder is the perpendicular distance between its bases.



Surface area of a cylinder

The **area** of the **curved surface** of a **cylinder = circumference of the base × altitude**

Surface area of a **cylinder = area of curved surface + area of the base + area of top**
∴ **Surface area of a cylinder** with **base radius** $\boldsymbol{r}$ and **altitude** $\boldsymbol{h = 2\pi rh + 2\pi r^2}$**.**

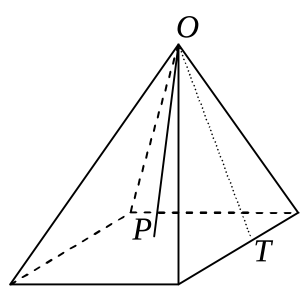
Volume of a cylinder

Volume of a **cylinder** = **area of base** × **altitude**
∴ The **volume of a cylinder** with **base radius** $\boldsymbol{r}$ and **altitude** $\boldsymbol{h = \pi r^2 h}$**.**

Pyramid

A **pyramid** is a solid bounded by plane faces, of which one, called the base, is any rectilinear figure, and the rest (lateral faces) are triangles having a common vertex O at some point not in the plane of the base. The perpendicular height from the vertex to the base is the **altitude** of the pyramid.
A pyramid having a **regular polygon** for its base is said to be **right** when the vertex lies on a straight line drawn perpendicular to its base, from the central point. When the base is regular the lateral faces are all equal isosceles triangles. The **slant height** of a regular pyramid is the altitude of one of the lateral faces.

Right pyramid with a square base



OP – altitude, OT – slant height

Surface area of a pyramid

The **lateral area** $\boldsymbol{S}$ of a **right pyramid** with a **regular base** is equal to **one half the product of its slant height** $\boldsymbol{l}$ and **base perimeter** $\boldsymbol{p}$; $\boldsymbol{S} = \frac{1}{2}(\boldsymbol{p} \times \boldsymbol{l})$.

The **surface area** $\boldsymbol{A}$ of a **right pyramid** with a **regular base** is the **sum** of **the lateral area** $\boldsymbol{S}$ and the **area** $\boldsymbol{B}$ **of the base;** $\boldsymbol{A = S + B}$

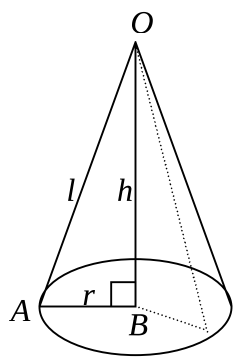
Volume of a pyramid

The **volume** $\boldsymbol{V}$ of a **right pyramid with a regular base** is equal to **one third the product of the base area** $\boldsymbol{B}$ and **altitude** $\boldsymbol{h}$**;** $\boldsymbol{V = \frac{1}{3}(B \times h)}$**.**

Cones

A **right circular cone** is a solid described by the revolution of a right-angled triangle about one of the sides containing the right-angle as axis.

Thus if the right-angled triangle *ABO* revolves about *OB* which remains fixed, the hypotenuse *OA* describes the curved surface of the cone represented in the diagram below. The point *O* is called the **apex** of the cone. The **altitude** is the length *h* of *OB*, and the **slant height** is the length *l* of the hypotenuse *OA*. The circle described by the radius *AB* is the **base of the cone**.



Surface area of a right circular cone

A**rea** of the **curved surface** of the **cone =** $\frac{1}{2}$ **(circumference of the base × slant height)**

Surface area of the cone = area of curved surface + area of the base

$\therefore$ **Surface area of a cone** with **base radius *r* and slant height *l*** $= (\frac{1}{2} \times 2\pi r \times l) + \pi r^2$

$$= \pi r\,(l + r)$$

Volume of a cone

Volume of a cone = $\frac{1}{3}$**(area of base × altitude)**

$\therefore$ **The volume *V* of a cone** with **base radius *r*** and **altitude *h*** is given by the formula $V = \frac{1}{3}\pi r^2 h$.

Sphere

A **sphere** is a solid contained by **one curved surface** which is such that all points on it are equidistant from a fixed point within it called the **centre**. Any line drawn from the centre to the surface is a **radius** of the sphere.

Surface area of a sphere

The **surface area** $\boldsymbol{A}$ of a **sphere** of **radius** $\boldsymbol{r}$ is given by the formula $\boldsymbol{A = 4\pi r^2}$

Volume of a sphere

The **volume** $\boldsymbol{V}$ of a **sphere** of **radius** $\boldsymbol{r}$ is given by the formula $\boldsymbol{V = \frac{4}{3}\pi r^3}$

Example:

(i) Find the surface area and the volume of a hemisphere bowl 1 cm in thickness and 10 cm in external radius.

(ii) Find the radius of a sphere with surface area equal to the surface area of a cylinder with a top, of altitude 16 cm and base diameter 4 cm.

(iii) An iron sphere of diameter 6 cm is dropped into a cylindrical vessel partially filled with water. The diameter of the vessel is 12 cm. If the sphere is completely immersed, by how much will the surface of the water be raised?

Solution:

(i) The surface area of the outer hemisphere = $[\frac{1}{2}\times 4\times \pi \times (10)^2]cm^2 = 200\pi\ cm^2$

The surface area of the inner hemisphere = $[\frac{1}{2}\times 4\times \pi \times (9)^2]cm^2 = 162\pi\ cm^2$

The area of the rim = $[\pi(10)^2 - \pi(9)^2]cm^2 = 19\pi\ cm^2$

Thus the total surface area of the bowl is $(200\pi + 162\pi + 19\pi)\,cm^2 = 381\pi\ cm^2$.

The volume of the bowl is

$$[\frac{1}{2}\times\frac{4}{3}\times\pi\times(10)^3 - \frac{1}{2}\times\frac{4}{3}\times\pi\times(9)^3]\,cm^3 = \frac{2}{3}\pi(1000-729)\,cm^3 = \frac{542}{3}\pi\ cm^3.$$

(ii) The surface area of the cylinder is $[2\times\pi\times(2)^2 + 2\times\pi\times 2\times 16]\,cm^2 = 72\pi\ cm^2$.

If the radius of the sphere is r cm, then $4\pi r^2 = 72\pi$. Thus $r^2 = 18$ and hence $r = 4.24$ (to the second decimal place). Thus the radius of the sphere is approximately 4.24 cm.

(iii) If the surface of the water is raised by h cm, then $\frac{4}{3}\times\pi\times(3)^3 = \pi\times(6)^2\times h$.

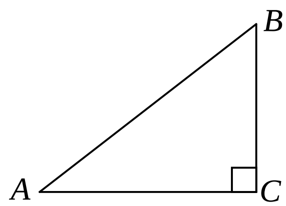
Thus $h = 1$ cm.

4.3 Pythagoras' Theorem

Pythagoras' theorem is an important theorem in geometry. It has many application in mathematics, in other fields of study as well as in real life situations.

Pythagoras' Theorem

In any right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides



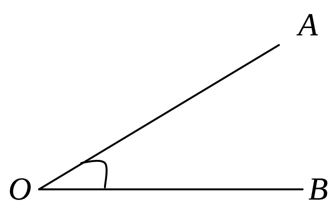
$$AB^2 = AC^2 + BC^2$$

4.4 Introduction to trigonometry

The word **trigonometry** means triangle measurement.

4.4.1 Angles

When two straight lines meet they form an angle.



The angle between the straight lines OA and OB is denoted by $A\hat{O}B$.

The size of the angle $A\hat{O}B$ is a measure of how far the line OA has been rotated from a starting position along OB. Two lines are said to be at **right angles**, if the rotating line starting from one position to another describes one quarter of a circle. If the rotation is in an anti-clockwise direction, the angle is said to be **positive**. If not, it is said to be **negative**.

To measure angles, a particular angle is fixed and is taken as a **unit** of measurement so that any other angle is measured by the number of times it contains the unit.

4.4.2 Sexagesimal System

In the **sexagesimal system**, a **right angle** is divided into 90 equal parts called **degrees**. Each degree is divided into 60 equal parts called **minutes** and each minute is further sub-divided into 60 equal parts called **seconds**.

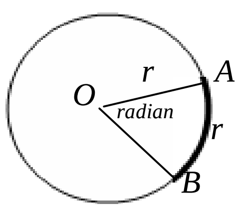
Thus
1 right angle = 90 degrees
1 degree = 60 minutes
1 minute = 60 seconds

In symbols a degree, a minute and a second are denoted respectively by 1^{o}, $1'$ and $1''$.

The unit of measurement in this system is the degree.

4.4.3 Circular System

In this system the unit of measurement is the radian. It is defined as the angle subtended at the centre of a circle by an arc equal to the radius of the circle.



The symbol 1^{c} is used to denote a radian.

Theorem: *The circumference of a circle bears a constant ratio to its diameter*.

The constant ratio in the above theorem is denoted by π.

$$\therefore \frac{\textbf{\textit{Circumference}}}{\textbf{\textit{Diameter}}} = \pi$$

An approximate value for π is the fraction $\frac{22}{7}$ or the decimal 3.14.

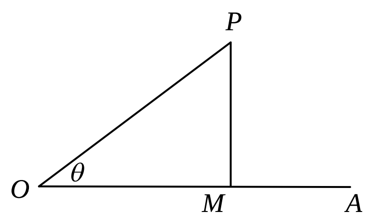
Relationship between the two systems

$$\boxed{\pi \text{ radians} = 180^{o}}$$

Note: Generally the superscript ‘c’ is omitted. Thus if the unit in terms of which the angle is measured is not mentioned, it should be assumed that the unit is radians.

4.4.4 Trigonometric ratios for angles less than a right angle

Let a revolving line *OP* start from *OA* and revolve into the position *OP*, tracing out an angle $A\hat{O}P$. Let PM be the perpendicular drawn from *P* to the line *AO*.



In the triangle *POM*,
OP is the **hypotenuse**,
PM is the **perpendicular** and
OM is the **base**.

The **trigonometric ratios** of the angle θ ($A\hat{O}P$) are defined as follows:

$\frac{MP}{OP} = \frac{\text{Perp}}{\text{Hyp}}$ is called the **sine** of the angle θ and is written as $\sin\theta$

$\frac{OM}{OP} = \frac{\text{Base}}{\text{Hyp}}$ is called the **cosine** of the angle θ and is written as $\cos\theta$

$\frac{MP}{OM} = \frac{\text{Perp}}{\text{Base}}$ is called the **tangent** of the angle θ and is written as $\tan\theta$

$\frac{OM}{MP} = \frac{\text{Base}}{\text{Perp}}$ is called the **cotangent** of the angle θ and is written as $\cot\theta$

$\frac{OP}{MP} = \frac{\text{Hyp}}{\text{Perp}}$ is called the **cosecant** of the angle θ and is written as $\operatorname{cosec}\theta$

$\frac{OP}{OM} = \frac{\text{Hyp}}{\text{Base}}$ is called the **secant** of the angle θ and is written as $\sec\theta$

Note: It follows from the definitions that

(i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (ii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$ (iii) $\cot\theta = \frac{1}{\tan\theta}$

(iv) $\sec\theta = \frac{1}{\cos\theta}$ (v) $\cos ec\theta = \frac{1}{\sin\theta}$

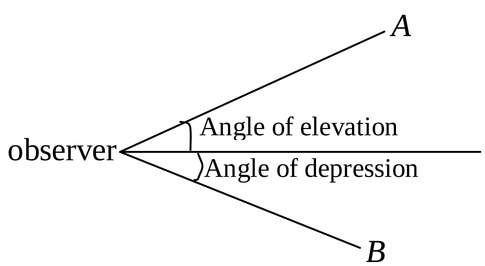
Values of some trigonometrical ratios

Angle	0^o	30^o	45^o	60^o	90^o
Sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

4.4.5 Problems involving angles of elevation and angles of depression

One of the important applications of trigonometry is determining distances between points and heights of objects without actually measuring them.

When a person observes an object, the angle formed between the horizontal and the line of sight is called
the **angle of elevation**, if the object lies above the horizontal,
the **angle of depression** if the object lies below the horizontal.

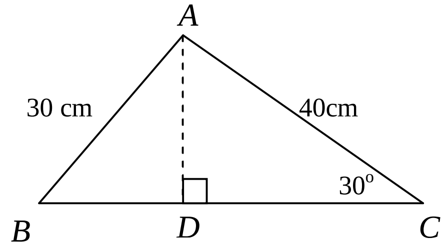


Example:

(i) From the top of a tree of height 20m, the angles of depression to the top and bottom of a house are 30^o and 45^o respectively. Determine the height of the house and the distance from the tree to the house.

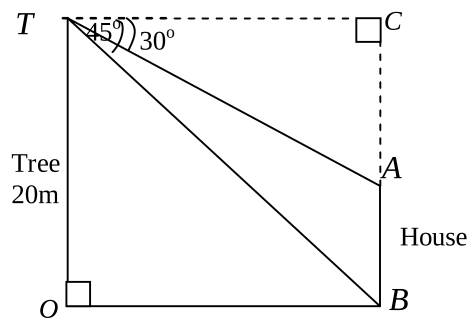
(ii) The distance to a tower of height 25 m from point *B* is 40 m. If points *A* and *B* lie on a straight line leading to the tower and the angle of elevation to the top of the tower from point *B* is twice the angle of elevation from point *A*, determine the distance from point *A* to point *B*.

(iii) Determine *BC* in the given figure.



Solution:

(i)



From the figure we obtain $CB = TO = 20$ m.

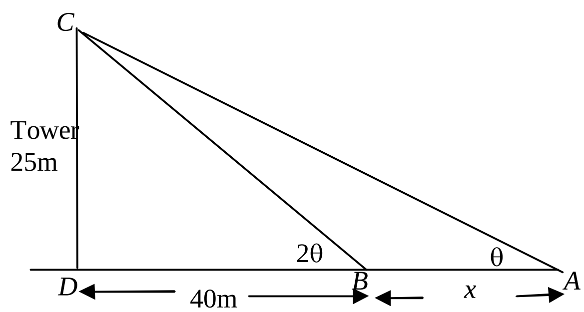
Considering triangle TCB we obtain $\tan 45^{o} = \frac{CB}{TC}$.

Since $\tan 45^{o} = 1$, $TC = CB = 20$m.
But $OB = TC$.
Therefore the distance from the tree to the house is 20m.

$\tan 30^{o} = \frac{CA}{TC}$.

Therefore $CA = TC \times \tan 30^{o} = 20 \times 0.5774$.
i.e., $CA = 11.55$ m (to two decimal places).
Thus the height of the house $AB = CB - CA = 8.45$ m

(ii)



Let x denote the distance from point A to point B.

From triangle BCD, $\tan 2\theta = \frac{25}{40} = 0.6250$.

Thus $2\theta = 32^{o}$ (from the Table of Natural Tangents).
Therefore, $\theta = 16^{o}$.

Considering triangle ACD we obtain

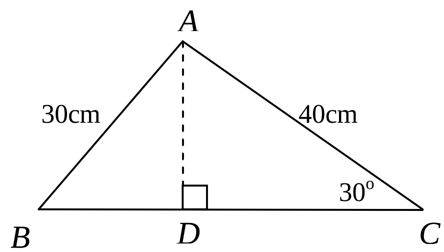
$\tan 16^{o} = \frac{25}{40+x}$.

$\tan 16^{o} = 0.2867$ (from the Table of Natural Tangents)

Therefore, $40 + x = \dfrac{25}{\tan 16} = \dfrac{25}{0.2867} = 87.20$ (to two decimal places).

Thus $x = 47.20$ m.

(iii)



$\sin 30^{\circ} = \dfrac{AD}{40}$. Therefore, $AD = 40 \times \dfrac{1}{2} = 20$ cm

$\cos 30^{\circ} = \dfrac{DC}{40}$. Therefore, $DC = 40 \times \dfrac{\sqrt{3}}{2} = 20\sqrt{3}$ cm

By Pythagoras' theorem, $AB^2 = AD^2 + BD^2$

Therefore, $BD^2 = 900 - 400 = 500$

Hence $BD = 10\sqrt{5}$ cm.

Therefore, $BC = 20\sqrt{3} + 10\sqrt{5}$ cm.